

# Neutrino masses from the GSI anomaly

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We investigate the influence of the strong Coulomb field of a heavy nucleus on massive neutrinos, produced in the K-shell electron capture (*EC*) decays of the H-like  $^{140}\text{Pr}^{58+}$  and  $^{142}\text{Pm}^{60+}$  ions. The corrections to the neutrino masses due to virtually produced charged lepton *W*-boson pairs in the strong Coulomb field of a nucleus with charge *Ze* are calculated and discussed with respect to their influence on the period of the time-modulation of the number of daughter ions, observed recently in the *EC*-decays of the H-like  $^{140}\text{Pr}^{58+}$  and  $^{142}\text{Pm}^{60+}$  ions at GSI in Darmstadt. These corrections explain the 2.9 times higher difference of the squared neutrino masses obtained from the time-modulation of the *EC*-decays with respect to the value deduced from the antineutrino-oscillation experiments of KamLAND. The values of neutrino masses are calculated.

PACS: 12.15.Ff, 13.15.+g, 23.40.Bw, 26.65.+t

The experimental investigation of the *EC*-decays of the H-like ions  $^{140}\text{Pr}^{58+}$  and  $^{142}\text{Pm}^{60+}$ , i.e.  $^{140}\text{Pr}^{58+} \rightarrow ^{140}\text{Ce}^{58+} + \nu_e$  and  $^{142}\text{Pm}^{60+} \rightarrow ^{142}\text{Nd}^{60+} + \nu_e$ , carried out in the Experimental Storage Ring (ESR) at GSI in Darmstadt [1], showed a modulation in time with periods  $T_{EC} \simeq 7$  s of the rate of the number of daughter ions. Since the rate of the number of daughter ions is defined by

$$\frac{dN_d^{EC}(t)}{dt} = \lambda_{EC}(t) N_m(t), \quad (1)$$

where  $\lambda_{EC}(t)$  is the *EC*-decay rate and  $N_m(t)$  is the number of mother ions  $^{140}\text{Pr}^{58+}$  or  $^{142}\text{Pm}^{60+}$ , the time-modulation of the rate of  $N_d^{EC}(t)$  implies a periodic time-dependence of the *EC*-decay rate  $\lambda_{EC}(t)$ .

As has been proposed in [2], such a periodic dependence of the *EC*-decay rate can be explained by the mass-differences of the neutrino mass-eigenstates. The period of the time-modulation

$T_{EC}$  has been obtained as

$$T_{EC} = \frac{4\pi\gamma M_m}{\Delta m_{21}^2}, \quad (2)$$

where  $M_m$  is the mass of the mother ion,  $\gamma = 1.43$  is the Lorentz factor of the H-like ions [1] and  $\Delta m_{21}^2 = m_2^2 - m_1^2$  is the difference of the squared neutrino masses  $m_2$  and  $m_1$ .

For  $T_{EC} = 7.06(8)$  s [1], measured for the H-like  $^{140}\text{Pr}^{58+}$  ion, we have got  $(\Delta m_{21}^2)_{\text{GSI}} = 2.18(3) \times 10^{-4} \text{ eV}^2$  [2], which is by a factor 2.9 larger than  $(\Delta m_{21}^2)_{\text{KL}} = 7.59(21) \times 10^{-5} \text{ eV}^2$  [3], used also for the global analysis of the solar-neutrino and KamLAND experimental data [4] (see also [5]). For the first time the value  $(\Delta m_{21}^2)_{\text{GSI}} \simeq 2.25 \times 10^{-4} \text{ eV}^2$  has been obtained by Kleinert and Kienle within the neutrino-pulsating vacuum approach [6]. The same estimate for  $\Delta m_{21}^2$  one can get by using the period of the time-modulation derived by Lipkin [7].

For the understanding of such a discrepancy we propose the following mechanism. In the *EC*-decay of a H-like heavy ion a daughter ion with electric charge *Ze* and a massive neutrino are produced. Since a characteristic energy scale

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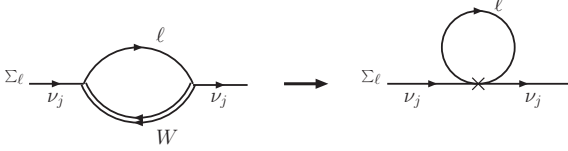


Figure 1. Feynman diagrams, defining corrections to the mass of a massive neutrino in a strong Coulomb field of a nucleus with charge  $Ze$ .

is of order of a few  $10^{-15}$  eV [2], one possible solution of the discrepancy between  $(\Delta m_{21}^2)_{\text{GSI}}$  and  $(\Delta m_{21}^2)_{\text{KL}}$  is that a massive neutrino gets a correction to its mass, caused by its interaction with the strong Coulomb field of the daughter ion due to virtually produced  $\ell^- W^+$  pairs, where  $\ell^- = e^-, \mu^-, \tau^-$  is a negatively charged lepton and  $W^+$ -boson, as an intermediate state. The Feynman diagrams of the process are depicted in Fig. 1 with the Green functions of virtual charged leptons in the strong Coulomb field [8,9].

For the calculation of the diagrams in Fig. 1 we use the weak leptonic interaction [10]

$$\begin{aligned} \mathcal{L}_W(x) = & -\frac{G_F}{\sqrt{2}} \sum_{j\ell} \sum_{\ell'j'} U_{j\ell} U_{\ell'j'}^* \\ & \times [\bar{\psi}_{\nu_j}(x) \gamma^\mu (1 - \gamma^5) \psi_\ell(x)] \\ & \times [\bar{\psi}_{\ell'}(x) \gamma_\mu (1 - \gamma^5) \psi_{\nu_{j'}}(x)], \end{aligned} \quad (3)$$

defined by the  $W$ -boson exchange, where  $x = (t, \vec{r})$ ,  $G_F$  is the Fermi constant,  $\psi_{\nu_j}(x)$  and  $\psi_\ell(x)$  are operators of the neutrino  $\nu_j$  and lepton fields  $\ell = e^-, \mu^-, \tau^-$ , respectively, and  $U_{\ell j}$  are the elements of the unitary neutrino-flavour mixing matrix  $U$  [4]. In our analysis neutrinos  $\nu_j$  ( $j = 1, 2, 3$ ) are Dirac particles with masses  $m_j$  ( $j = 1, 2, 3$ ), respectively [4].

A correction  $\delta m_j$  to the neutrino mass, induced by the interaction of the neutrino  $\nu_j$  with a strong Coulomb field of a nucleus, is defined by

$$\delta m_j(r) = \sum_{\ell} U_{j\ell} U_{\ell j}^* \mathcal{M}_{\ell}(r), \quad (4)$$

where we have denoted

$$\mathcal{M}_{\ell}(r) = i\sqrt{2}G_F \int_{-i\infty}^{+i\infty} \frac{dE}{2\pi} \text{tr}\{G_{\ell}(\vec{r}, \vec{r}; E) \gamma^0\}. \quad (5)$$

Here  $G_{\ell}(\vec{r}, \vec{r}; E)$  is the energy-dependent Green function of the negatively charged leptons  $\ell^-$  in a strong Coulomb field, produced by a positive electric charge  $Ze$  [8,9].

Using the results, obtained in [9], we get

$$\begin{aligned} \mathcal{M}_{\ell}(r) = & \sqrt{2}G_F \frac{m_{\ell}}{\pi^2 r^2} \sum_{n=1}^{\infty} n \int_0^{\infty} \int_0^{\infty} dx dt \\ & \times e^{-2m_{\ell} r \sqrt{x^2+1} \coth t} \left\{ 2Z\alpha \coth t \cos\left(\frac{2Z\alpha x t}{\sqrt{x^2+1}}\right) \right. \\ & \times \tilde{I}_{2\nu}\left(\frac{2m_{\ell} r \sqrt{x^2+1}}{\sinh t}\right) - \sin\left(\frac{2Z\alpha x t}{\sqrt{x^2+1}}\right) \\ & \times \left[ \frac{2m_{\ell} r x}{\sinh t} \tilde{I}_{2\nu+1}\left(\frac{2m_{\ell} r \sqrt{x^2+1}}{\sinh t}\right) + \frac{2\nu x}{\sqrt{x^2+1}} \right. \\ & \left. \left. \times \tilde{I}_{2\nu}\left(\frac{2m_{\ell} r \sqrt{x^2+1}}{\sinh t}\right) \right] \right\}, \end{aligned} \quad (6)$$

where  $\nu = \sqrt{n^2 - (Z\alpha)^2}$  and  $I_{\mu}(z)$  is a modified Bessel function [11],  $\tilde{I}_{2\nu+1}(z) = I_{2\nu+1}(z) - I_{2n+1}(z)$  and  $\tilde{I}_{2\nu}(z) = I_{2\nu}(z) - I_{2n}(z)$ . We would like to notice that at  $Z\alpha \rightarrow 0$  the corrections to the neutrino masses vanish as  $\mathcal{M}_{\ell}(r) \rightarrow 0$ . Hence, a non-vanishing correction to the massive neutrino mass appears only due to the Coulomb field. Since at  $r \rightarrow \infty$  the corrections introduced by Eq.(5) vanish rapidly, so that in the subsequent interactions [12] the neutrino  $\nu_j$  should be with a proper mass  $m_j$ . The very rapid vanishing (see Fig. 2) of the  $\delta m_j(r)$  with  $r$  makes it reasonable to take into account the influence of the correction only at the nuclear surface [13] just after the production of the massive neutrino  $\nu_j$  and the daughter ion<sup>5</sup>.

At the nuclear radius  $r = R = 5.712$  fm [14],

<sup>5</sup>The amplitude of the  $EC$ -decay is proportional to [2,14]

$$\begin{aligned} M(m \rightarrow d\nu_j)(t) \propto & \int d^3x \Psi_d^*(r) \Psi_m(r) \psi_{1s}^{(Z)}(r) e^{iE\nu_j(r)t} = \\ = & e^{iE\nu_j(R)t} \langle \psi_{1s}^{(Z)} \rangle \mathcal{M}_{\text{GT}}, \end{aligned}$$

where  $E\nu_j(r) = \sqrt{k_j^2 + (m_j + \delta m_j(r))^2}$ . Using an analogy between the Fermi-Dirac distribution function and the Woods-Saxon shape of the nuclear density [15] and following [16] one can show that the relation  $\langle \psi_{1s}^{(Z)} e^{iE\nu_j t} \rangle = \langle \psi_{1s}^{(Z)} \rangle e^{iE\nu_j(R)t}$  is fulfilled with an accuracy better than 1%. For the confirmation of the validity of this relation we refer also on [13].

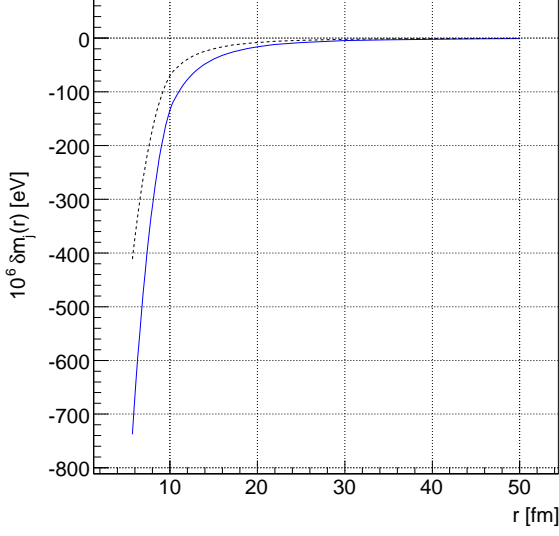


Figure 2. The corrections to the neutrino masses, caused by a strong nuclear Coulomb field, where  $\delta m_1(r)$  and  $\delta m_2(r)$  are presented by the solid and dotted line, respectively.

we get

$$\begin{aligned}\mathcal{M}_{e^-}(R) &= -2.02 \times 10^{-3} \text{ eV}, \\ \mathcal{M}_{\mu^-}(R) &= -5.16 \times 10^{-4} \text{ eV}, \\ \mathcal{M}_{\tau^-}(R) &= -3.88 \times 10^{-5} \text{ eV}\end{aligned}\quad (7)$$

with an electron  $e^-$ , muon  $\mu^-$  and  $\tau^-$ -lepton in the intermediate state, respectively. The corrections to the neutrino masses are equal to

$$\begin{aligned}\delta m_1(r) &= \cos^2 \theta_{12} \mathcal{M}_{e^-}(r) + \sin^2 \theta_{12} (\cos^2 \theta_{23} \\ &\quad \times \mathcal{M}_{\mu^-}(r) + \sin^2 \theta_{23} \mathcal{M}_{\tau^-}(r)), \\ \delta m_2(r) &= \sin^2 \theta_{12} \mathcal{M}_{e^-}(r) + \cos^2 \theta_{12} (\cos^2 \theta_{23} \\ &\quad \times \mathcal{M}_{\mu^-}(r) + \sin^2 \theta_{23} \mathcal{M}_{\tau^-}(r)),\end{aligned}\quad (8)$$

where  $\theta_{12}$  and  $\theta_{23}$  are mixing angles. The corrections to the neutrino masses Eq.(8) are defined for  $\theta_{13} = 0$  [2] (see also [5]). Then, setting  $\theta_{12} = 34^\circ$  and  $\theta_{23} = 45^\circ$  [5] we obtain

$$\begin{aligned}\delta m_1(R) &= -14.74 \times 10^{-4} \text{ eV}, \\ \delta m_2(R) &= -8.22 \times 10^{-4} \text{ eV}.\end{aligned}\quad (9)$$

The period of modulation is thus redefined as

$$T_{EC} = \frac{4\pi\gamma M_m}{(m_2 + \delta m_2(R))^2 - (m_1 + \delta m_1(R))^2}. \quad (10)$$

Neglecting the contributions of  $(\delta m_j(R))^2$  we transcribe the denominator into the form

$$\begin{aligned}\delta m_2^2(R) - \delta m_1^2(R) &= (\Delta m_{21}^2)_{\text{GSI}} - (\Delta m_{21}^2)_{\text{KL}} \\ &\quad + (\delta m_1(R))^2 - (\delta m_2(R))^2,\end{aligned}\quad (11)$$

where  $\delta m_j^2(R) = 2m_j\delta m_j(R)$ . Using the numerical values of the corrections Eq.(9),  $(\Delta m_{21}^2)_{\text{GSI}} = 2.20 \times 10^{-4} \text{ eV}^2$ ,  $(\Delta m_{21}^2)_{\text{KL}} = 7.59 \times 10^{-5} \text{ eV}^2$  and a relation  $m_2 - m_1 = (\Delta m_{21}^2)_{\text{KL}}/(m_2 + m_1)$  we solve Eq.(11) and get the following values for neutrino masses

$$\begin{aligned}m_2 &= 0.11 + 0.82 \times 10^{-4} \text{ eV}, \\ m_1 &= 0.11 + 4.26 \times 10^{-4} \text{ eV}.\end{aligned}\quad (12)$$

The mass  $m_3$  of the neutrino  $\nu_3$  is

$$m_3 = 0.12 + 8.05 \times 10^{-4} \text{ eV}. \quad (13)$$

We obtain it using Eq.(12) and the experimental value  $\Delta m_{32}^2 = 2.4 \times 10^{-3} \text{ eV}^2$  [10]. The sum of neutrino masses amounts to

$$\sum_{j=1,2,3} m_j = 0.34 \text{ eV}, \quad (14)$$

which agrees well with the upper limit  $\sum_j m_j < 1 \text{ eV}$  [4].

We have shown that an interaction of virtually produced  $\ell^- W^+$  pairs  $\nu_j \rightarrow \sum_\ell U_{j\ell} \ell^- W^+$  of massive neutrinos  $\nu_j$  in the strong Coulomb field of the daughter ion can induce certain corrections to neutrino masses, which allow to reconcile the value  $(\Delta m_{21}^2)_{\text{GSI}} = 2.18(3) \times 10^{-4} \text{ eV}^2$  [2], deduced from the period of the time-modulation of the rate of the number of daughter ions in the  $EC$ -decays of the H-like ions  $^{140}\text{Pr}^{58+}$  and  $^{142}\text{Pm}^{60+}$ , with  $(\Delta m_{21}^2)_{\text{KL}} = 7.59(21) \times 10^{-5} \text{ eV}^2$  [4,5], obtained as a best-fit of the global analysis of the solar-neutrino and KamLAND experimental data [4] (see also [5]). We would like to notice that for the calculation of the corrections to neutrino masses we have taken into account the contribution of the  $W^+$ -boson exchange only. The contribution of the  $Z$ -boson exchange is proportional to the constant  $g_V = -0.040 \pm 0.015$

[4]. This means that the corrections to neutrino masses, caused by the  $Z$ -boson exchanges, are smaller compared with corrections, which can be caused by the experimental uncertainties of the mixing angles  $\theta_{12} = 33.9^{+2.4}_{-2.2}$  degrees and  $\theta_{23} \leq 45$  degrees [4].

The proposed change of the neutrino masses together with the experimental data on the time-modulation of the rate of the number of daughter ions in the  $EC$ -decays of the H-like ions and  $(\Delta m_{21}^2)_{\text{KL}} = 7.59 \times 10^{-5} \text{ eV}^2$  allows to estimate the values of neutrino masses  $m_j \simeq 0.11 \text{ eV}$  agreeing well with the constraint on the sum of neutrino masses  $\sum m_j < 1 \text{ eV}$  [4]. The value of the heaviest neutrino mass  $m_3 = 0.12 + 8.05 \times 10^{-4} \text{ eV}$  satisfies also the constraint  $0.04 < m_3 < 0.40 \text{ eV}$  [4].

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